Student Number: ______________________________ Class Teacher: _________________________

St George Girls High School

Trial Higher School Certificate Examination

2018

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided at the back of this paper

Section II Pages 6 – 14

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section
- Begin each question in a new writing booklet

Section I	**/10**
Section II	
Question 11	/15
Question 12	/15
Question 13	/15
Question 14	/15
Question 15	/15
Question 16	/15
Total	**/100**

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

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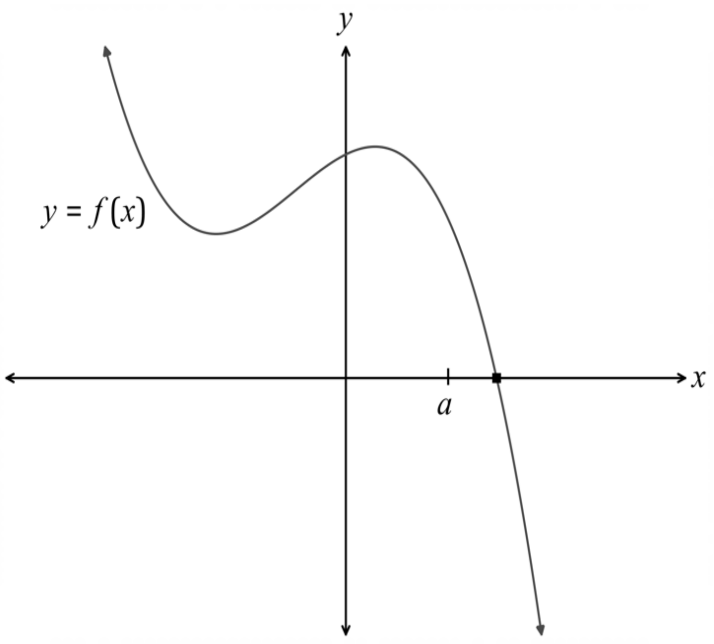
1. The first three terms of an arithmetic sequence are 2, 6 and 10.
   What is the 15th term of this sequence?

   (A) 58
   (B) 62
   (C) 450
   (D) 480

2. The diagram below shows the graph of $y = f(x)$.



   Which of the following statements is true for $x = a$?

   (A) $f'(a) < 0$ and $f''(a) > 0$
   (B) $f'(a) > 0$ and $f''(a) > 0$
   (C) $f'(a) < 0$ and $f''(a) < 0$
   (D) $f'(a) > 0$ and $f''(a) < 0$

Section I (cont'd)

3. What is the area enclosed between $y = -\sqrt{1 - x^2}$ and the x-axis from $x = -1$ to $x = 1$?

(A) $\frac{1}{2}\pi$

(B) π

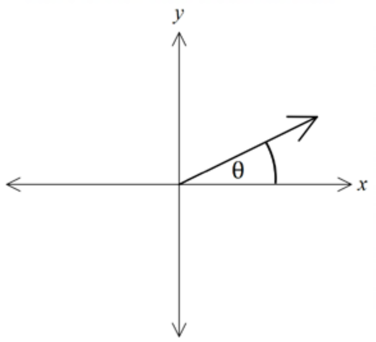
(C) 2π

(D) 4π

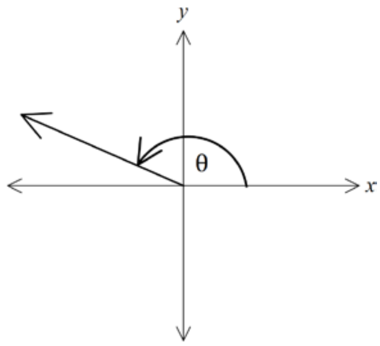
4. For the angle θ, $\sin\theta = -\frac{8}{17}$ and $\tan\theta = -\frac{8}{15}$.

Which diagram best shows angle θ?

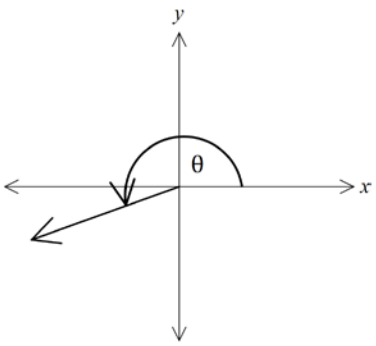
(A)



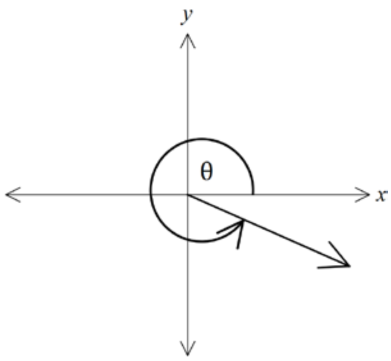
(B)



(C)



(D)



5. Find the limiting sum of the series $5 + \frac{5}{7} + \frac{5}{49} + \cdots$

(A) ∞

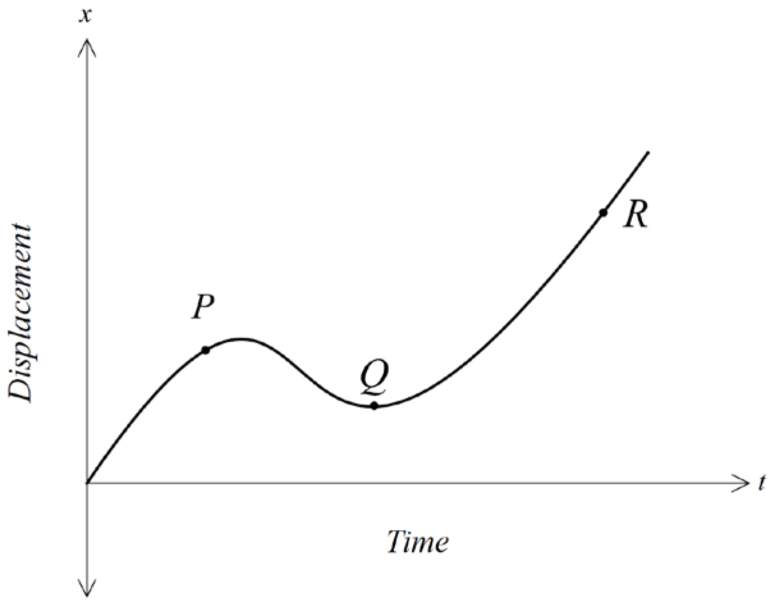
(B) $\frac{5}{42}$

(C) $5\frac{5}{6}$

(D) 6

Section I (cont'd)

6. The graph shows the displacement x of a particle moving along a straight line as a function of time t.



Which statement correctly describes the motion of the particle?

(A) At point P, its acceleration and velocity are both positive.

(B) At point P, its acceleration is negative while its velocity is positive.

(C) At point Q, the particle is stationary and its acceleration is zero.

(D) At point R, the particle is stationary and its acceleration is zero.

7. Find $\int \frac{1}{4x+1}\,dx$.

(A) $\frac{-4}{(4x+1)^2} + C$

(B) $4\ln(4x+1) + C$

(C) $\frac{1}{4}\ln(4x+1) + C$

(D) $\ln(4x+1) + C$

Section I (Continued).

8. What is the derivative of $x + \ln x$?

(A) $1 + \dfrac{1}{x}$

(B) $1 + \ln x$

(C) $\dfrac{x^2}{2} + \dfrac{1}{x}$

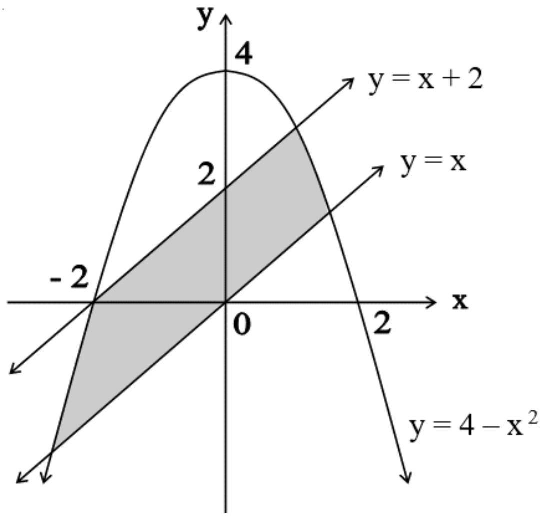
(D) $1 + \dfrac{1}{\ln x}$

9. Given that $\int_a^b f(x)\,dx = k$ and $\int_b^a g(x)\,dx = k - 2$.

What is the value of $\int_a^b [f(x) + g(x)]\,dx$?

(A) 2
(B) -2
(C) $2k - 2$
(D) $2 - 2k$

10. The shaded region in the diagram is enclosed by $y = 4 - x^2$, $y = x$ and $y = x + 2$.



Which of the following is the set of inequations that satisfy the shaded region?

(A) $y \geq 4 - x^2, y \leq x, y \leq x + 2$
(B) $y \leq 4 - x^2, y \leq x, y \leq x + 2$
(C) $y \leq 4 - x^2, y \leq x, y \geq x + 2$
(D) $y \leq 4 - x^2, y \geq x, y \leq x + 2$

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section.

Start each question in a new writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a New Writing Booklet. **Marks**

a) Factorise $a^2 + a - 12$ 1

b) Find the value of r given that $\dfrac{\sqrt{5}}{\sqrt{5}-2} = 5 + r\sqrt{5}$. 2

c) Find $\int (\sin x + \cos x)\, dx$. 1

d) Differentiate $y = x \tan x$. 2

e) Solve $|x - 5| > 2$. 2

f) Find the coordinates of the focus of the parabola $(x+3)^2 = 12(y-1)$. 2

g) Find the domain of the function $f(x) = \ln(9 - x)$. 1

h) Find the equation of the tangent to the curve $y = x^3 - 2x$ at the point $(1, -1)$. 2

i) Find $\displaystyle\int_0^1 (1 + e^{-x})\, dx$. 2

Question 12 (15 marks) Start a New Writing Booklet. **Marks**

a) In the diagram below, the points $A(2,0)$, $B(4,3)$ and $C(3,4)$ are shown.

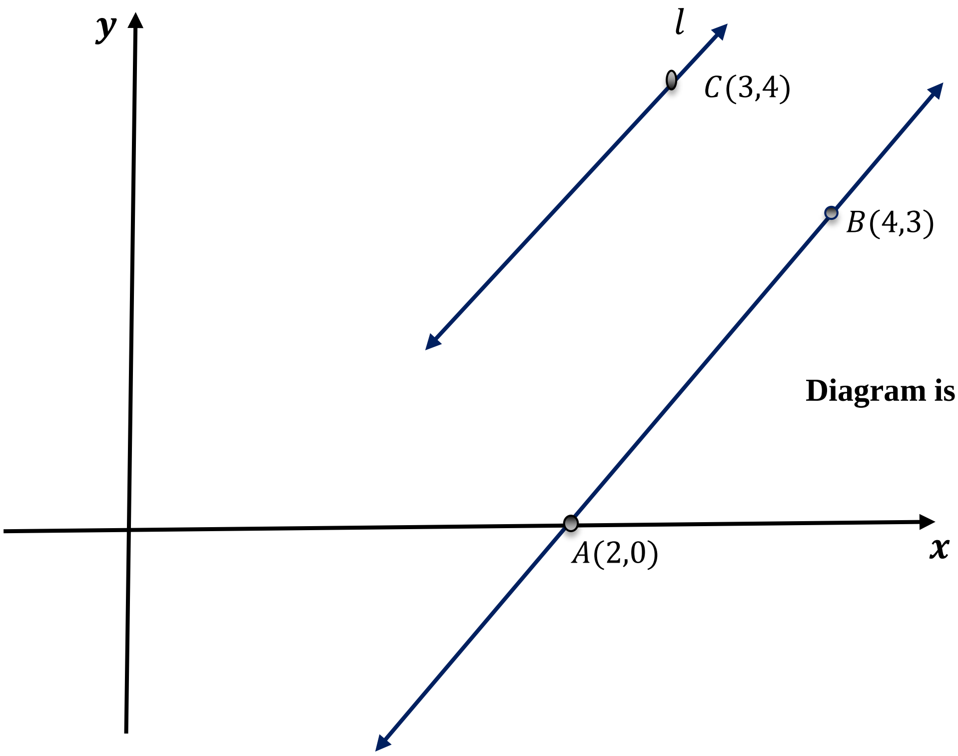


Diagram is not drawn to scale.

(i) What is the exact length of AB ? 1

(ii) Show that the equation of AB is $3x - 2y - 6 = 0$. 1

(iii) Find the exact perpendicular distance from C to AB. 1

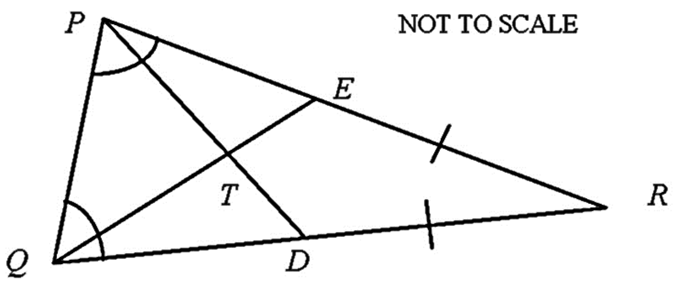
(iv) The line l passing through C has the equation $3x - 2y - 1 = 0$.
(DO NOT PROVE THIS).
Show that the line l is parallel to AB. 2

(v) D is a point on the line l such that the length DC is $\frac{\sqrt{13}}{2}$ units.
What type of quadrilateral is $ABCD$?
Give a reason for your answer. 1

(vi) Calculate the area of $ABCD$. 1

Question 12 (Continued). **Marks**

b) In the diagram $ER = DR$ and $\angle QPR = \angle PQR$. QE and PD intersect at T.

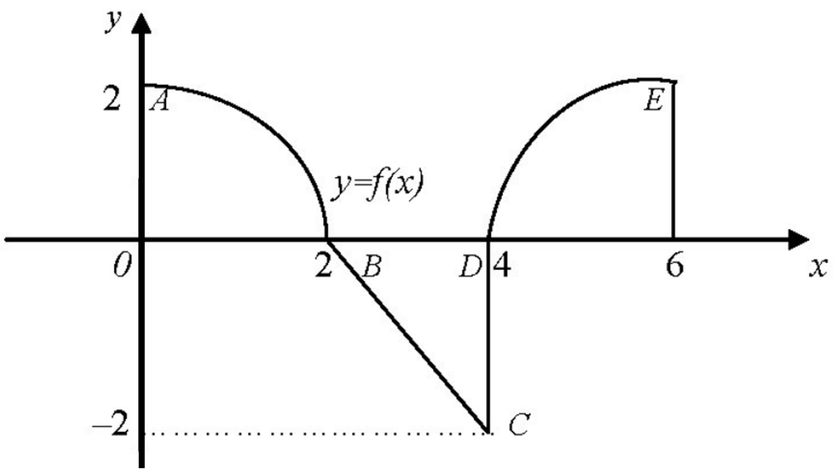


(i) Copy this diagram into your writing booklet.

Prove, with full reasoning, that ΔQEP is congruent to ΔPDQ. 3

(ii) Why is ΔQTP is isosceles? 2

c) The graph of $y = f(x)$ consists of a quarter circle AB, triangle BCD and quarter circle DE as shown in the diagram below.



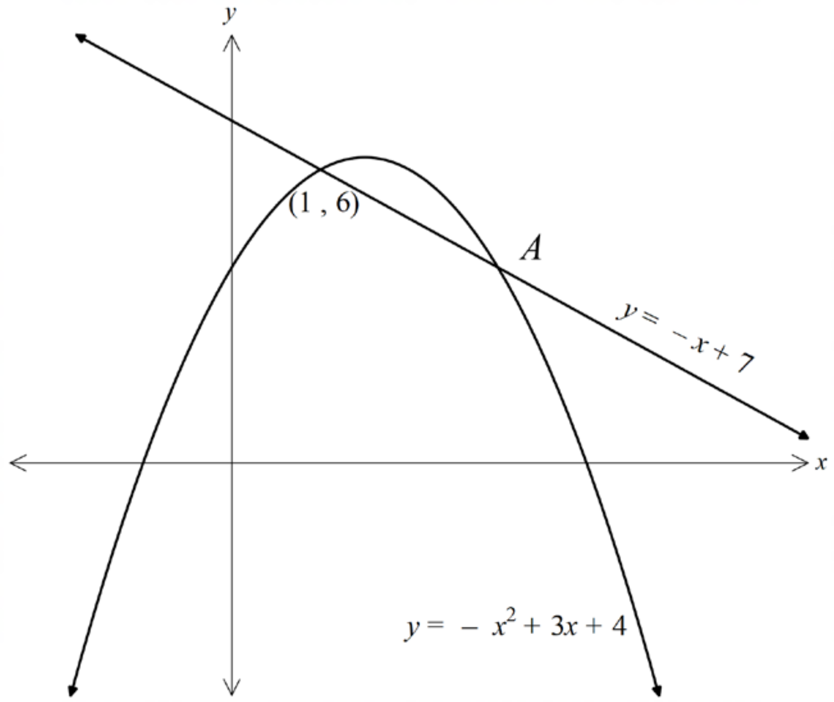
(i) Evaluate $\int_0^6 f(x)\ dx$. 2

(ii) State the values of x satisfying $0 < x < 6$ to indicate where $y = f(x)$ is *not* differentiable. 1

Question 13 (15 marks) Start a New Writing Booklet. **Marks**

a) Draw a neat sketch of $y = 3\sin 4x$ for $0 \leq x \leq \pi$. 2
Show clearly all of the relevant features.

b) The parabola $y = -x^2 + 3x + 4$ and the line $y = -x + 7$ intersect at the point $(1, 6)$ and at point A.



(i) Show that the x-coordinate of point A is 3. 2

(ii) Calculate the area enclosed by the parabola and the line. 3

c) Consider the function $f(x) = x^3 - 3x^2 - 24x + 20$.

i) Find the coordinates of the turning points of the curve and determine their nature. 3

ii) Find the values of x for which the curve is decreasing. 1

iii) Sketch the curve showing the turning points and the y –intercept. 2

d) Solve the equation $2(\ln x)^2 - \ln x - 1 = 0$.
Leave your answer in exact form. 2

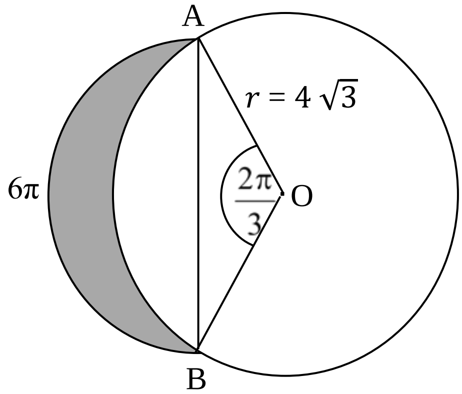
Question 14 (15 marks) Start a New Writing Booklet. **Marks**

a) Solve $2\cos 2\theta = 1$, for $0 \le \theta \le 2\pi$. 2

b) In the diagram, AB is a chord of a circle with centre O and radius $r = 4\sqrt{3}$, such that $\angle AOB = \dfrac{2\pi}{3}$.

AB is also the diameter of a semicircle with arc length 6π cm.

The length of $AB = 12$ cm.



Find the shaded area ,**in exact form,** which lies inside the semicircle, but outside the circle. 3

c) At a particular location, a river 24 metres wide is measured for depth every 6 metres across its width. The measurements from bank to bank are given in the following table:

Distance across the river (m)	0	6	12	18	24
Depth (m)	0	8	22	6	0

(i) Use Simpson's rule to find the cross-sectional area of the river at this point. 2

(ii) Use your answer in part (i) to find the volume of water passing through this point in 3 hours, if the water passing this point travels at $\frac{1}{4} m/sec$. 1

d) If α and β are the roots of $5x^2 - 2x + 6 = 0$, find the values of:

(i) $\alpha + \beta$. 1

(ii) $(\alpha + 1)(\beta + 1)$. 1

Question 14 (Continued). **Marks**

e) The initial size of a new bee colony was registered at 120000.
The number of bees B, in the colony was represented by $B = B_0e^{0.5t}$, where t is in hours.

(i) How many more bees were added to the colony during the first 5 hours? 2

(ii) How fast was the colony growing at the 5-hour mark? 2

(iii) How long did it take for the new colony to double in size? 1

Question 15 (15 marks) Start a New Writing Booklet. **Marks**

a) Evaluate $\lim_{x \to 2} \left(\frac{x^3 - 8}{x - 2} \right)$. 2

b) Find all values of k, if $(k-2), \sqrt{3k}, (k+2)$ form a geometric progression. 3

c) Evaluate $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x}$. 2

d) A particle moves along the x-axis with velocity $v(t)$ metres per second given by $v(t) = 16t - t^3$ after time t seconds.
At time $t = 2$, the displacement of the particle was 15 metres to the right of the origin.

i) Write an expression for the acceleration $a(t)$ of the particle. 1

ii) Find an expression for the displacement $s(t)$ of the particle. 2

iii) Find the total distance travelled between the times $t = 2$ and $t = 6$. 1

e) i) Prove 2

$$\frac{1}{1 - cos\theta} + \frac{1}{1 + cos\theta} = 2cosec^2\theta.$$

ii) Hence or otherwise solve 2

$$cosec\theta \left[\frac{1}{1 - cos\theta} + \frac{1}{1 + cos\theta} \right] = 16\text{ , for } 0^\circ \leq \theta \leq 2\pi.$$

Question 16 (15 marks) Start a New Writing Booklet. **Marks**

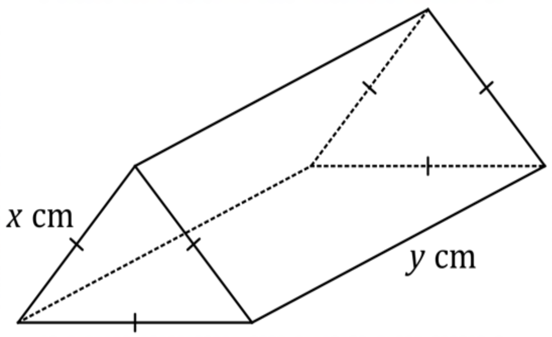
a) A large tank of liquid which contains L litres of a toxic chemical is being drained. The amount of chemical in the tank over time t minutes, is given by:

$$L = 110(20 - t)^2.$$

i) At what rate is the water draining out of the tank after 5 minutes? 2

ii) How long will it take for the tank to be completely empty? 1

b) A triangular prism has a base that is an equilateral triangle with a side length of x cm. The length of the triangular prism is y cm. The volume of the prism is 1000 cm^3.



Given the expression of y in terms of x is:

$y = \dfrac{4000}{x^2\sqrt{3}}$ (DO NOT PROVE THIS)

i) Show that the surface area, A cm^2 of the prism is given by: 2

$$A = \frac{4000\sqrt{3}}{x} + \frac{\sqrt{3}x^2}{2}$$

ii) What is the value of x for the prism that will minimise the surface area? 3

Question 16 (Continued). **Marks**

c) The region bounded by $y = p^2 - x^2$ and the x–axis, where p is a constant, is rotated about the y–axis between $y = 0$ to $y = p^2$ to form a solid.

Find the volume of this solid, in terms of p. 2

d) Jane borrows \$60 000 to buy a new car.

The interest rate charged on the loan is 0.08% per week compounding weekly.

She agrees to repay the loan in equal fortnightly repayments of \$1000 each.

Let A_n be the amount of in dollars owing after her n^{th} fortnightly repayment.

i) Show that $A_2 = \$60\ 000\ \times 1.008^4 - 1000\ (1 + 1.0008^2)$. 1

ii) Using part (i), or otherwise, show that $A_n = \$624\ 750.1 - \$564\ 750.1 \times 1.0008^{2n}$. 2

iii) How many weeks would it take Jane to repay her loan? 2

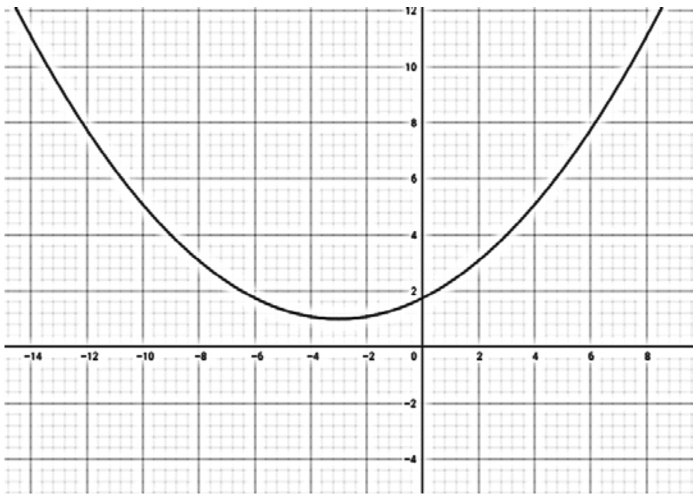
END OF THE EXAMINATION

Section 1

Question	Solution	Marking Guideline
1	*a* = 2 and *d* = 4 $Tn = a + (n-1)d$ $T_{15} = 2 + (15-1) \times 4$ $= 58$	1 Mark: A
2	When *x* = *a* the curve is decreasing and concave down. $\therefore f'(a) < 0$ and $f''(a) < 0$	1 Mark: C
3	The required area is a semicircle with a radius of 1 unit. $A = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times 1^2$ $= \frac{1}{2}\pi$ square units	1 Mark: A
4	sin and tan are both negative in the fourth quadrant	1 Mark: D
5	$5 + \frac{5}{7} + \frac{5}{49} + \cdots$ $\frac{a}{1-r} = \frac{5}{1-\frac{1}{7}} = 5\frac{5}{6}$	1 Mark: C
6	At point P, the slope of the curve is positive, therefore the velocity is positive. Concavity is negative, so acceleration is negative.	1 Mark: B
7	$\int \frac{1}{4x+1}\,dx$ $= \frac{1}{4}\int \frac{4}{4x+1}\,dx$ $= \frac{1}{4}\ln(4x+1) + C$	1 Mark: C
8	As $y = x + \ln x$, then $\frac{dy}{dx} = 1 + \frac{1}{x}$.	1 Mark: A
9	If $\int_b^a g(x)\,dx = k - 2$ then $\int_a^b g(x)\,dx = 2 - k$ Hence, $\int_a^b \left(f(x) + g(x) \right) dx = k + 2 - k = 2$	1 Mark: A
10	Select a point to test in the inequalities to decide which are correct. Test (0, 0) in $y \le 4 - x^2$: $0 \le 4 - 0$, true Test (0, 0) in $y \le x + 2$: $0 \le 0 + 2$, true Test (0, 1) in $y \le x$: $1 \le 0$, false, so $y \ge x$ Hence, option C contains the 3 correct inequalities.	1 Mark: D

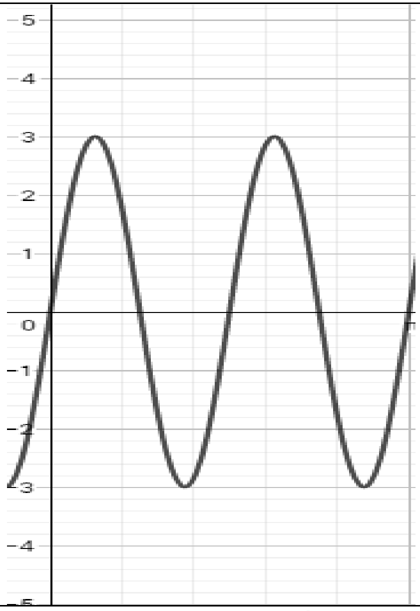
Section 2

Q11	Solution	Marking Guidelines
a	$a^2 + a - 12$ $= a^2 + 4a - 3a - 12$ $= a(a+4) - 3(a+4)$ $= (a-3)(a+4)$	1 Mark for correct factors.
b	$\frac{\sqrt{5}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$ $= \frac{5+2\sqrt{5}}{5-4}$ $= 5 + 2\sqrt{5}$ So $r = 2$	1 Mark for rationalising the denominator . 1 Mark for correct value of *r*.
c	$\int (\sin x + \cos x)\, dx$ $= -\cos x + \sin x + C$	1 Mark for correct integration.
d	Using the product rule: $Let\, u = x$ and $v = tan\, x$ $u' = 1$ and $v' = sec^2 x$ $\frac{dy}{dx} = u'v + v'u$ $= \tan x \times 1 + x\, sex^2 x$ $= tan\, x + x\, sec^2 x$	1 Mark for correct application of the product rule. 1 Mark for appropriate simplification.
e	$\|x-5\| > 2$ $x - 5 > 2$ or $-x + 5 > 2$ $x > 7$ $-x > -3$ $x > 7$ $x < 3$	1 Mark for correct inequalities set up using concept of absolute value. 1 mark for correct solution.
f	The equation is in the form $(x-h)^2 = 4a(y-k)$ $(x+3)^2 = 12(y-1)$ Vertex $= (-3,1)$ $4a = 12$ $a = 3$	1 Mark for vertex and focal length.

	 Hence, the co-ordinates of the focus are (-3,4)	1 Mark for correct coordinates of the focus.
g	$f\ (x)\ = \ln(9-x)$ is only defined when $9-x>0$ $-x>-9$ $x<9$ The domain is: $x<9$ **or** all real $x : x<9$	1 Mark for the correct domain.
h	$y\ =\ x^3\ -\ 2x$ $\dfrac{d\,y}{dx}\ =\ 3x^2 - 2$ When $x\ =\ 1$, the tangent will have gradient $m = 3 \text{ x } 1 - 2$ $m = 1$ The equation of the tangent will be $y - y_1 = m(x - x_1)$ $y - (-1)\ =\ 1(x - 1)$ $y\ +\ 1\ =\ x - 1$ $y\ =\ x - 2$	1 Mark for differentiating and finding the gradient. 1 Mark for finding the equation of the tangent.
h	$\int_0^1 (1+e^{-x})dx = [x-e^{-x}]_0^1$ $= (1-e^{-1}) - (0-e^0)$ $= 1 - \frac{1}{e} + 1$ $= 2 - \frac{1}{e}$	1 Mark for correct integration. 1 Mark for appropriate substitution and simplification.

Q12	Solution	Marking Guidelines
a (i)	$AB = \sqrt{(4-2)^2 + (3-0)^2}$ $= \sqrt{13}$	1 Mark for correct answer.
a (ii)	$\frac{y-0}{x-2} = \frac{3-0}{4-2}$ $2y = 3x - 6$ $3x - 2y - 6 = 0$	1 Mark for correct equation.
a (iii)	$d = \left\|\frac{3(3) - 2(4) - 6}{\sqrt{3^2 + 2^2}}\right\|$ $= \left\|\frac{-5}{\sqrt{13}}\right\|$ $= \frac{5}{\sqrt{13}}$ or $\frac{5\sqrt{13}}{13}$	1 Mark for correct perpendicular distance.
a (iv)	*AB*: $2y = 3x - 6$ $y = \frac{3}{2}x - 3$ $m_1 = \frac{3}{2}$ or $m_1 = -\frac{a}{b} = \frac{3}{2}$ Similarly, for l: $2y = 3x - 1$ $y = \frac{3}{2}x - \frac{1}{2}$ $m_2 = \frac{3}{2}$ *or* $m_2 = -\frac{a}{b} = \frac{3}{2}$ ∴*AB*//line l: (gradients are equal)	1 Mark for finding the gradient of *AB* or line *M*. 1 mark for correct justification.
a (v)	*ABCD* is trapezium. (One pair of opposite sides are parallel, but not equal)	1 Mark for correct reasoning.
a(vi)	$A = \frac{1}{2}(a+b)h$ $= \frac{1}{2}\left(\frac{\sqrt{13}}{2} + \sqrt{13}\right) \times \frac{5}{\sqrt{13}}$ $= \frac{15}{4}$ or 3.75 square units	1 Mark for correct area.
b(i)	In Δ*QEP* and Δ*QDP*, *PR* = *QR* (sides opposite equal angles *ER*=*DR* (given) ∴*PE* = *QD* (by subtraction of sides) *QP* is common < *QPE* = < *PQD* (given) ∴ Δ*QEP* ≡ Δ*QDP* (SAS holds)	1 Mark for justification of subtraction of sides. 1 Mark for the other 2 reasons. 1 Mark for appropriate congruency test.
b(ii)	Δ*TEP* ≡ Δ*QDT* , < *QTD* = < *PTE* (vertically opposite angles are equal) < *QDT* = < *PET* (corresponding angles in congruent triangles,	1 Mark for appropriate reasoning.

	$\Delta QEP \equiv \Delta QDP$) $QD = PE$ (already proven) Since $\Delta TEP \equiv \Delta TDQ$ (AAS) $PT = QT$ (corresponding sides of congruent triangles) $\therefore \Delta QTP$ is isosceles (2 equal sides) Or $\angle EQP = \angle DPQ$ (corresponding angles in congruent triangles) $\therefore \Delta TPQ$ $is\ isosceles(angles\ opposite\ equal\ sides\ are\ equal)$	1Mark for appropriate justification of an isosceles triangle.
c (i)	$\int_0^6 f(x)\ dx =$ ¼ circle – triangle + ¼ circle = ¼ π (4) – ½ (2x2) + ¼ π (4) = $2\pi - 2$	1 Mark for appropriate working. 1 Mark for correct value of the integral.
c (ii)	$x = 2\ and\ x = 4$	1 Mark for the appropriate points.

Q13	Solution	Marking Guidelines
a	 Amplitude = 3 Period = $\frac{\pi}{2}$	1 Mark for correct shape. 1 Mark for correct amplitude and period. Some students were confused between the sine and cosine curve.
b (i)	$y = -x^2 + 3x + 4$ ① $y = -x + 7$ ② $-x^2 + 3x + 4 = -x + 7$ (*sub* ① into (2)) $-x^2 + 4x - 3 = 0$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $\therefore$ $x = 1$ (which we already knew) and $x = 3$ $\therefore$ point A has x-coordinate 3.	1 Mark for correct use of simultaneous equations. 1 Mark for correct x −coordinate. Most students failed to set up simultaneous equations appropriately.
b (ii)	$A = \int_1^3 (-x^2 + 3x + 4) - (-x + 7) \; dx$ $= \int_1^3 (-x^2 + 4x - 3) \; dx$ $= \left[-\frac{x^3}{3} + 2x^2 - 3x \right]_1^3$ $= \left[\left(-\frac{27}{3} + 2(3^2) - 3(3) \right) - \left(-\frac{1}{3} + 2(1^2) - 3(1) \right) \right]$ $= \left[0 + \frac{4}{3} \right]$ $= \frac{4}{3}$ square units.	1 Mark for setting up correct integral. 1 Mark for correct integration. 1 Mark for correct answer. The majority of students attempted this question very well.

c (i)

$f(x) = x^3 - 3x^2 - 24x + 20$

$f'(x) = 3x^2 - 6x - 24$

Let $f'(x) = 0$ to find the possible stationary turning points, we get:

$3x^2 - 6x - 24 = 0$ that is $x^2 - 2x - 8 = 0$

so $(x-4)(x+2) = 0$ Hence, x = 4 or x = – 2

When $x = 4$, y = – 60 and when x = –2, y = 48

In the table,

x	-3	–2	0	4	5
$f'(x)$	21	0	−24	0	21
$f(x)$	↗	(–2, 48) max	↘	min (4, – 60)	↗

$f'(-3) > 0$ and $f'(0) < 0$

So the curve has a maximum at (−2, 48).

Also, as $f'(0) < 0$ and $f'(5) > 0$ then the curve has a minimum at (4, – 60).

Alternative method:

$$f''(x) = 6x - 6$$

$$f''(-2) = -18$$

Hence max turning point at (−2, 48).

$$f''(4) = 18$$

Hence min turning point at (4, – 60).

Marks: 1 Mark for correct differentiation and making the expression equal to zero.

1 Mark for determining the stationary points.

1 Mark for determining the nature of the stationary points.

The majority of students attempted this question very well.

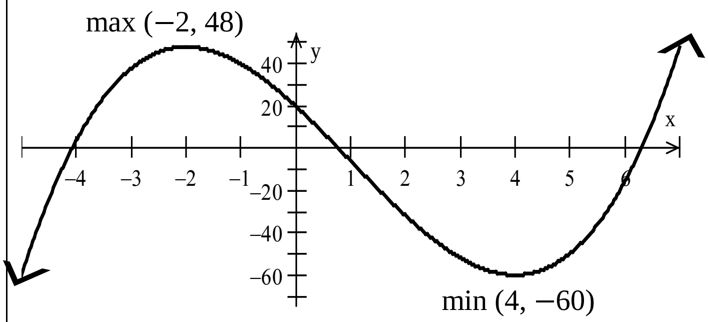
c (ii)

The curve is decreasing when $f'(x) < 0$.

Hence, it is decreasing for $-2 < x < 4$

Marks: 1 Mark for correct values of x.

c (iii)



Marks: 1 Mark for correct shape.

1 Mark for displaying the turning point and *y*-intercept.

Well done.

d

$2\ln^2 x - \ln x - 1 = 0$

Let k = ln x we get $2k^2 - k - 1 = 0$

$(2k + 1)(k - 1) = 0$

So $k = -\frac{1}{2}$ or $k = 1$

Hence $\ln x = -\frac{1}{2}$ or $\ln x = 1$

$x = e^{-\frac{1}{2}}$ $\left(ie\ \frac{1}{\sqrt{e}}\right)$ or $x = e$

Note both answers are valid as they verify the original equation.

Marks: 1 Mark for correct values of *k*.

1 Mark for the correct values of *x*.

Some students were confused with the log being negative compared to positive log.

Q14	Solution	Marking Guidelines
a	$2\cos 2\theta = 1$ $\cos 2\theta = \frac{1}{2}$ $2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	1 Mark for half the correct solutions. 1 Mark for the remaining solutions.
b	The shaded region = area of semicircle with diameter AB – area of minor segment with chord AB. $= \frac{1}{2} \times \pi \times 6^2 - \frac{1}{2} \times (4\sqrt{3})^2 \times (\frac{2\pi}{3} - \sin\frac{2\pi}{3})$ $= 18\pi - 24 \times (\frac{2\pi}{3} - \frac{\sqrt{3}}{2})$ $= 18\pi - 16\pi + 12\sqrt{3}$ $= (2\pi + 12\sqrt{3})$ cm ^{2}	2 Marks for appropriate calculation. 1 Mark for the correct answer.
c (i)	$A \approx \frac{h}{3}[y_0 + 2(y_2 + y_4 + \cdots) + 4(y_1 + y_3 + \cdots) + y_l]$ $A \approx \frac{6}{3}[0 + 2(22) + 4(8 + 6) + 0]$ $A \approx 200\ m^2$	1 Mark for correct application of Simpson's Rule. 1 Mark for correct area.
c (ii)	Volume = $200\ m^2 \times \frac{1}{4}\ m/s \times 3hrs \times 3600s$ Volume = $540\ 000\ m^3$	1 Mark for correct answer.
d (i)	$5x^2 - 2x + 6 = 0$ $\alpha + \beta = -\frac{b}{a} = \frac{2}{5}$	1 Mark for correct answer.
d (ii)	$(\alpha + 1)(\beta + 1)$ $= \alpha\beta + \alpha + \beta + 1$ $= \frac{6}{5} + \frac{2}{5} + 1 = \frac{13}{5}$	1 Mark for correct answer.
e (i)	B = $B_0 e^{0.5t}$ So B = $120000e^{0.5t}$ B size = $120000e^{0.5(5)}$ At t=5 B = 1461899 approximately. Number added = 1461899 – 120000 = 1341899 bees	1 Mark for correct substitution and attaining 1461899. 1 Mrk for the number added to the colony.
e (ii)	(b) (ii) $\frac{dB}{dt} = 60000e^{0.5(5)} = 730949.6$	1 Mark for correct Differentiation 1 Mark for finding the correct rate.
	(iii) $240000 = 120000e^{0.5t}$ $2 = e^{0.5t}$ $ln2 = 0.5t$ $t = 2\ ln2 = ln4 = 1.39$ hrs	1 Mark for correct substitution and evaluation of time.

Q15	Solution	Marking Guidelines
a	$\lim_{x\to 2}\left(\frac{x^3-8}{x-2}\right)$ $=\lim_{x\to 2}\left(\frac{(x-2)(x^2+2x+4)}{x-2}\right)$ $=\lim_{x\to 2}(x^2+2x+4)$ $=(2)^2+2(2)+4$ $=12$	1 Mark for factorising difference of cubes. 1 Mark for correct limit.
b	$\frac{k+2}{\sqrt{3k}}=\frac{\sqrt{3k}}{k-2}$ $k^2-4=3k$ $k^2-3k-4=0$ $(k-4)(k+1)=0$ $k=-1,4$ $k>0$ $\therefore k=4$	1 Mark for using the correct test with correct substitution. 1 Mark for achieving $k=-1,4$. 1 Mark for correct solution.
c	$\int_0^{\frac{\pi}{4}}\frac{dx}{\cos^2 x}=\int_0^{\frac{\pi}{4}}\sec^2 x\ dx$ $=\left[\tan x\right]_0^{\frac{\pi}{4}}$ $=1-0$ $=1$	1 Mark for achieving $\int_0^{\frac{\pi}{4}}\sec^2 x\,.\,dx.$ 1 Mark for correct solution.
d (i)	$\frac{dv}{dt}=a=16-3t^2$	1 Mark for achieving $a=16-3t^2$
d (ii)	$s(t)=\int(16t-t^3)\ dt$ $s=15\ ,t=2$ $=8t^2-\frac{1}{4}t^4+c$ $15=32-4+c$ $\therefore\ c=-13$ $So\quad s(t)=8t^2-\frac{1}{4}t^4-13$	1 Mark for achieving $=8t^2-\frac{1}{4}t^4+c$ 1 Mark for achieving $s(t)=8t^2-\frac{1}{4}t^4-13$
d (iv)	from t=2 to t=4 travels 36 metres t=6 travels $\lvert-100\rvert$ $\therefore$ total distance = 136	1 Mark for appropriate Working and answer.

e(i)	$\frac{1}{1-cos\theta}+\frac{1}{1+cos\theta}$ $=\frac{1+cos\theta+1-cos\theta}{(1-cos\theta)(1+cos\theta)}$ $=\frac{2}{(1-cos\theta)(1+cos\theta)}$ $=\frac{2}{1-\cos^2\theta}$ $=\frac{2}{\sin^2\theta}$ $=2cosec^2\theta$	1 Mark for obtaining $\frac{2}{(1-cos\theta)(1+cos\theta)}$ 1 Mark for further simplification
e(ii)	$\text{cosec}\theta\left[\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}\right]=16$ $\text{cosec}\theta\left(2\cos ec^2\theta\right)=16$ $\text{cosec}^3\theta=8$ $\text{cosec}\theta=2$ $\sin\theta=\frac{1}{2}$ $\theta=\frac{\pi}{6},\frac{5\pi}{6}$	1 Mark for obtaining $\text{cosec}^3\theta=8$ 1 Mark for correct values of θ.

<table>
<tr><th>Q16</th><th>Solution</th><th>Marking Guidelines</th></tr>
<tr><td>a(i)</td><td>$L = 110(20-t)^2$
$\therefore \text{Rate} = \frac{dL}{dt} = 110 \times 2(20-t) \times -1$
$= -220(20-t)$
$\therefore \text{at } t = 5$
$\frac{dL}{dt} = -220(20-5) = -3300\, L/Min$</td><td>1 Mark for correct differentiation.

½ mk for 3300
½ mk for the units
Ignored the sign as the tank was draining.</td></tr>
<tr><td>a(ii)</td><td>For empty tank : $L = 0$
i.e. $110(20-t)^2 = 0$
$(20-t)^2 = 0$
$\therefore t = 20$ minutes

Or use $\frac{dL}{dt} = 0$</td><td>1 Mark for $L = 0$ and t=20 minutes.</td></tr>
<tr><td>b (i)</td><td>$A = 3xy + 2\left(\frac{1}{2} \times x \times x \times \sin 60^\circ\right)$
$= 3xy + 2\left(\frac{\sqrt{3}}{4}x^2\right)$
$= 3x \times \frac{4000}{x^2\sqrt{3}} + 2\left(\frac{\sqrt{3}}{4}x^2\right)$
$= \frac{4000\sqrt{3}}{x} + \frac{\sqrt{3}x^2}{2}$

Or
$V = 1000$
$1000 = Area\ of\ triangle \times y$
$Area\ of\ triangle = \frac{1000}{y}$
$Surface\ Area = \frac{2000}{y} + 3xy, etc$</td><td>1 Mark for finding the correct expression for the surface area.

Or use Pythagoras' Theorem

1 Mark for the correct simplification.</td></tr>
<tr><td>b (ii)</td><td>Minimal A occurs when $\frac{dA}{dx} = 0$
$A = \frac{4000\sqrt{3}}{x} + \frac{\sqrt{3}x^2}{2}$
$\frac{dA}{dx} = -4000\sqrt{3} \times x^{-2} + \sqrt{3}x$
Hence
$-4000\sqrt{3} \times x^{-2} + \sqrt{3}x = 0$
$\frac{4000\sqrt{3}}{x^2} = \sqrt{3}x$</td><td>1 Mark for finding the derivative.</td></tr>
</table>

	$x = \sqrt[3]{4000}$ Check if a minima $\frac{d^2A}{dx^2} = 8000\sqrt{3} \times x^{-3} + \sqrt{3}$ When $x = \sqrt[3]{4000}$ $\frac{d^2A}{dx^2} = 8000\sqrt{3} \times \left(\sqrt[3]{4000}\right)^{-3} + \sqrt{3}$ $= 2\sqrt{3} + \sqrt{3} > 0$, therefore minimum.	1 Mark for finding the value of *x* without testing for a minima. 1/2 Mark **for justifying** whether the solution is maxima or minima. ½ mark for the answer (including 5.196…)
c	Volume of rotation about the y axis is: $V = \pi \int_a^b x^2\, dy$ $V = \pi \int_0^{p^2} (p^2 - y)\, dy$ $= \pi \left[p^2 y - \frac{y^2}{2} \right]_0^{p^2}$ $= \pi \left[(p^4 - \frac{p^4}{2}) - (0 - 0) \right] = \frac{\pi p^4}{2}$ units^3	1 Mark for correct application of the volume formula. 1 Mark for correct answer. Note the $\int p^2 dy = p^2 y$
d (i)	$A_1 = \$60000 \times 1.0008^2 - 1000$ $A_2 = (\$60000 \times 1.0008^2 - 1000) \times 1.0008^2 - 1000$ $= \$60000 \times 1.0008^4 - 1000 \times 1.0008^2 - 1000$ $= \$60000 \times 1.0008^4 - 1000 \times (1 + 1.0008^2)$	1 Mark for showing the necessary steps.
d (ii)	$A_2 = \$60000 \times 1.0008^4 - 1000 \times (1 + 1.0008^2)$ $A_n = 60000 \times 1.0008^{2n} - 1000(1 + 1.0008^2 + \ldots + 1.0008^{2n-2})$ $= 60000 \times 1.0008^{2n} - 1000\left(\frac{(1.0008^2)^n - 1}{1.0008^2 - 1}\right)$ $= 60000 \times 1.0008^{2n} - 624750.1(1.0008^{2n} - 1)$ $= 60000 \times 1.0008^{2n} - 624750.1 \times 1.0008^{2n} + 624750.1$ $= 624\,750.1 - 564750.1 \times 1.0008^{2n}$	½ mark ½ mark ½ mark ½ for no errors leading to the answer. Note Many students did not use (1.0008^2) As the common ratio.

d (iii)	The loan will be repaid when $A_n = 0$ Solve: $0 = \$624750.1 - 564750.1 \times 1.0008^{2n}$ $564750.1 \times 1.0008^{2n} = 624750.1$ $1.0008^{2n} = 624750.1 \div 564750.1$ $1.0008^{2n} = 1.10624\ldots.$ $\ln(1.0008^{2n)} = \ln(1.10624\ldots.)$ $2n \times \ln(1.0008^{)} = \ln(1.10624\ldots.)$ $2n = \frac{\ln(1.10624\ldots.)}{\ln(1.0008)}$ $n = \frac{\ln(1.10624\ldots.)}{2\ln(1.0008)}$ n = 63.129 fortnights n = 126.26 weeks	1 Mark for appropriate substitution into the formula and simplification. 1/2 Mark for correct answer. ½ mark for correct time unit.